Organisational Structure for Mathematical Modelling

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The effects of reading comprehension on the mathematical-modelling problem-solving process is yet unresolved. This paper reports on a study conducted with two classes of year four students. It investigated the extent to which a literary organisational structuring strategy: top-level structure, may change students' engagement in mathematical modelling. Mathematical-modelling problems require students to negotiate various texts as they negotiate the problem-solving process to learn mathematical skills. The strategy of top-level structuring aids students to structurally organise textual information, and to elicit and recall the main idea of texts. This research illustrated that top-level structuring can make a difference to young students' mathematical-modelling outcomes to some degree. Further in-depth research on the issues raised here is warranted.

We live in a technologically advancing society. Students need to be prepared thoroughly through the provision of learning opportunities that equip them with the necessary skills to operate effectively in this world. Mathematics and complex mathematical reasoning are essential components of these learning opportunities. The view taken here is that mathematical modelling is one way to provide students with such opportunities (English & Watters, 2005). As mathematical information is often embedded in complex textual material, high levels of literacy are critical to enable people to access numerical information and mathematical understandings. Top Level Structuring (TLS) has also proven to provide such an opportunity in various curriculum areas (Meyer, 2003). Skills like constructing, describing, explaining, manipulating and predicting complex systems such as business plans or budgeting plans, and understanding these systems (Lesh & Doerr, 2003; English, in press) are the skills that should form the crux of curriculum persuasion today, thus indicating the timeliness of this research.

Mathematical Modelling with Top-level Structure

Mathematical modelling provides a rich basis for empowering students and teachers with skills to function effectively in today's world. It features authentic problem situations in which students can explore and create models as possible solutions while investigating in a social context. These models form a basis for conceptualising the nature of modelling.

Mathematical-modelling problems allow for multi-interpretations and approaches to problem solution. The multifaceted end-products that children generate are to be shared in a social context and can be revised accordingly. English (2003) claimed that these features provide learning opportunities that encourage optimal development of mathematical skills. Furthermore, English and Lesh (2003) have emphasised that it is not just reaching the goal that is important, but also the interpretation of the goal, the information provided, and the possible steps to solution.

Mathematical modelling provides opportunities for students to acquire skills such as interpreting, thinking, communicating of ideas, justifying, revising, refining, and extending ideas while participating in a team of investigators to produce a model (Lesh & Doerr, 2003). When students participate in the process of mathematical modelling, they are participating in a process of interpretation of information from various text sources such as narrative texts, graphic texts like tables, diagrams or graphs and expository texts of facts and explanation. They must extract the main ideas, make assumptions, decide on their goal, explain their ideas, predict outcomes, and construct their case in an interactive social context. In doing so, students may employ other mathematical skills such as, number sense, measuring and comparing amounts. Additionally, they need to coordinate and organise all information gathered in their group (English, 2004; Lesh & Doerr, 2003; Lesh & Yoon, 2004).

In real-life, managers, human resource personnel, teachers, board and committee members and so on, get together in teams to address particular problem situations. Mathematical modelling provides opportunities for students to learn the skills to successfully operate in these situations.

Ultimately, mathematical modelling is about how a wide range of mathematics skills are learned and used. It offers "a rich platform for students' independent development of powerful math ideas" (Doerr & English, 2003, p. 122). The nature of mathematical modelling involves "multiple cycles of interpretation and re-interpretation of evolving products," therefore, there is no one approach to a solution. There is communication and sharing, describing, explaining, justifying and decision making. (Doerr & English, 2003). Mathematical-modelling problems are structured to promote open methodology for solving problems. The efficient employment of communicative skills suggests the need for strategic interpretation of the language of mathematics, strategic planning, sharing and justification of mathematical information. In other words, a strategic approach to mathematising "real life" problem situations could serve to enhance students' engagement in the mathematical process and their communication of the mathematical product resulting from active participation in model-eliciting tasks.

Top-level structure equates to the key structuring of the written symbolic language in a logical and systematic manner (Bartlett, 2003). The purpose of TLS is to help the reader or writer make sense of a situation by seeing the relationships present within the situation: that is, how an oral or written text is put together or structured to give meaning. In other words, TLS fosters thinking skills as students recognise, identify and classify structure (Bartlett, Barton, & Turner, 1989). They can use the structure to elaborate their thoughts, order and compare ideas, and to reflect, discern and infer from text. This in turn can enhance communication skills because giving structure to ideas enables their strategic delivery. TLS can be applied to any text, be it narrative, expository or graphic. Kiewra (2002) argues structural strategies are means of teaching students how to learn. TLS is an aid for reading and comprehension in the content areas including mathematics and as a result gain knowledge more efficiently. For example, through mathematical texts such as labelled tables, graphs or written information as found in mathematical-modelling problems, students could be aided to ascertain the author's main message through the application of structural thinking procedures which enhance ability to gain mathematical knowledge.

Bartlett and Fletcher (2001), determine four basic structures for TLS: comparison, cause/effect, problem/solution and listing/description. Applying a structure to text is simplified by the fact that texts contain signalling words. Examples of these are listed here in Table 1 (adapted from Meyer & Poon, 2001, p. 143). These give clues to the reader as to which structure is the best choice for a particular text.

Table 1: Organisational Structures in Text

TEXT STRUCTURE	SIGNALLING WORDS
Comparison	but, however, on the other hand, the same as, while,
Cause/Effect	as a result, because, since, if/then, so, therefore
Problem/Solution	problem, issue, solution, answer, reply, to solve this
List/Description	and, also, firstly, furthermore, for example, such as.

Bartlett (2003) reported that when students plan, they are more likely to interact and discuss how they extracted a main idea. The strategy also equips them with the ability to communicate effectively about content of text and act strategically upon the content by way of explanation, justification or argument.

Having strategic knowledge about how to use text effectively can serve to give students confidence and as a result encourage persistence with texts (Meyer, 2003). Skilled readers have been found to already structure text according to the author's textual organisation (Meyer, Brandt, & Bluth, 1980) but, gaining strategic knowledge can be especially significant for students who have difficulties in comprehending text.

The nature of modelling tasks requires students to employ high-level literacy skills with different types of texts so that they can engage fully with the problem. Interpreting textual information provides the basis through which students begin to gain mathematical knowledge (Lesh et al., 2003). To think mathematically requires interpretation and communication of problems at least as much as computation. Thinking mathematically is about constructing and making sense of complex systems like systems to forecast economic conditions (Lesh, Zawojewski, & Carmona, 2003).

Design and Methodology

My purpose in this research was to gauge the extent to which TLS may enhance mathematical modelling. I worked with two year four classes and implemented a teaching experiment (Cobb, 2000) where one class was initially taught the top-level structuring strategy (the TLS group) and the other class (the non-TLS) group was not taught the strategy for the implementation of the first mathematical-modelling problem. Before the implementation of the second mathematical-modelling problem the non-TLS group was also taught the TLS strategy. There was an initial comparison of the two classes after the first problem and then a final comparison after the two classes had been taught TLS. The part of the overall study that is reported here relates specifically to four groups of students, two from the TLS group and two from the non-TLS group. General references to the two classes as a whole are also made to amplify the sample. I report and discuss the results obtained after the implementation of the first mathematical-modelling problem.

Implementation of the First Modelling Activity

The activity discussed here was the first of two mathematical-modelling problems that the children completed. Neither group had ever been exposed to mathematical modelling prior to these experiences. Firstly, the TLS group was taught top-level structuring over a period of one school term. Following this, both the TLS group and non-TLS group participated in the mathematical-modelling problem ‘Beans, Glorious Beans’ shown in

Figure 1 (English & Watters, 2005).

Using the data, determine which of the light conditions is suited to growing Butter beans to produce the greatest crop. In a letter to Farmer Ben Sprout, **outline** your recommendation of light condition and **explain how** you arrived at this decision.

Predict the weight of butter beans produced on week 12 for each type of light. **Explain how** you made your prediction so Farmer Ben can use it in similar situations.

Sunlight Shade

Butter Bean Plants	Week 8	Week 9	Week 10	Butter Bean Plants	Week 8	Week 9	Week 10
Row 1	9 kg	12 kg	13 kg	Row 1	5 kg	9 kg	15 kg
Row 2	8 kg	11 kg	14 kg	Row 2	5 kg	8 kg	14 kg
Row 3	9 kg	14 kg	18 kg	Row 3	6 kg	9 kg	12 kg
Row 4	10 kg	11 kg	17 kg	Row 4	6 kg	10 kg	13 kg

Figure 1: Beans, Glorious Beans Problem

Data Collection and Analysis

The Year 3, 2004 test results for the participating students were used to provide background knowledge on students' mathematical and language capabilities, which provided part of the explanation for effectual issues of the learning processes of the study. (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). Data was collated from audio and video taping of selected student discussion groups and oral presentations, students' written work, and teacher/researcher observations. The tapes were transcribed and the following attributes between the groups were compared:

- The students' thinking processes (analysing the problem situations, planning solutions, explaining and justifying suggested actions, predicting their consequences, drawing together results and communicating these in forms that are meaningful and useful to others, critically evaluating one another's products, and responding productively to peer critiques.)
- The students' application of mathematics and literacy concepts (interpreting, and representing data, relating mathematical ideas, comprehending narrative, expository and graphic texts) to gain mathematical knowledge.

To ascertain the contribution of TLS to mathematical modelling in this comparison, the following research questions were addressed in the analysis:

- Was there evidence that students used an author's structure and/or key words to organize and subsequently express their ideas in written and oral presentations? e.g. "We recommend that Farmer Bean grows beans in the sunlight/shade because …."
- Was there evidence that structuring the text beneficially changed the way students could mathematize and gain mathematical knowledge through mathematical modelling?

Results

It is worth noting that in this particular year level, thirteen students out of fifty-three scored below the middle 50% of students tested in the state of Queensland in the Year 3 Aspects of Numeracy Test and a further five students were in the lower average range.. Notably seventeen students scored more than 15 percent below the state mean in 'Measurement and Data" and thirteen were in the lower average range. Eleven students scored in the lower 15% range in 'Number' with a further nine scoring in the lower average score range.

In reading and viewing, sixteen students were in the lower 15 percentile and nineteen were on or below the average score line. In writing, eighteen students were in the lower 15 percentile and nineteen scored below the average score line.

When students were initially presented with the problem, there was a noted difference. The TLS groups immediately began exploring the texts for an author's organisational structure and discussed what that might be before ascertaining the goal of the problem. The non-TLS groups immediately began questioning what they had to do. A number of these students asked, "So what do we have to do?" and needed qualification from the researcher or teacher.

A number of groups from both classes also began focussing on their prior knowledge of the best conditions in which to grow plants before starting to consider the mathematical data. It was observed that some students continued to fluctuate between the data and their prior knowledge on plants.

It was observed that in analysing the data shown in Figure 1, students in the focus groups mostly added the quantities of each row and compared the results. One group from the non-TLS group incorrectly summed all the data of all the rows thus comparing totals of 146kg and 112kg.

Overall, there was some evidence that students from the TLS group did use the author's plan of comparison and TLS key words in analysing the problem situation and planning their solution, for example:

> Ben: I reckon I would choose sunlight because if you compare with week 6, in the sunlight they got more kilograms.
>
> Students: It's a comparison.
>
> Researcher: What sort of a solution are you coming up with for the problem so far?
>
> Matthew: That shade has got less than the sunlight. We looked all through the weeks and the sunlight is getting more and more (kilos).
>
> Taylah: I think Farmer Sprout should plant beans in the sunlight because on the table it shows that there are more kilos in the sunlight than in the shade.

The mathematical part of the discussion in the non-TLS groups centred also on obtaining totals for the rows and comparing the totals. It appeared that although students did compare the totals, they were unaware of what they were actually doing because it was not evident in the language they used.

> Tim: I thought it was sunlight because look at this: On row 3, it's 18, row 3 in the shade it's only 12 kilograms,. Row 4 is 17 kilograms in the sunlight. In the shade it's only 13 kilograms.

And in the other non-TLS group:

Isobella: We worked out that sunlight is better than shade because you get more kilograms.

Kristy: We added up 9, 12, 13 …

In making the predictions regarding the growth of beans for week 12, all groups looked for patterns in the data and despite the anomalies they found still regarded that some sort of pattern existed as justification for their predictions. There was no evidence that TLS was considered for this investigation.

The students' letters in the TLS group used the language of comparison in expression more prevalently than the non-TLS group, for example:

> Dear Farmer Sprout, We arrived at this decision by reading the page of information and deciding it was comparison. The sunlight was similar to the shade but we chose sunlight because we recommend it is better for the beans…because they weighed more… If you look at our results in the sunlight week 6, row 1, you will see that the sunlight weighs more, has more beans and is taller than the shade one.
>
> Dear Farmer Sprout, we recommend as a group that you should plant your butter beans in the sunlight because on the chart sunlight has a bigger rate of kilograms than shade. We worked it out by doing a sum. (Students demonstrated sums of all the rows)

The non-TLS groups wrote:

> Dear Farmer Sprout, We think sunlight is the best choice of light because beans are supposed to grow in a warm place and in the sunlight the beans grow quicker. The beans are bigger and the beans are heavier, for example in week 6, row 1's sunlight was 9 kg and the shade had 5kg.
>
> To Farmer Sprout, We think you should plant your butter beans in the sun because it helps the beans produce more kilograms. It produces 146kg. We added the numbers up and it came to a bigger number than we would have thought and more than the shade as well.

When it came to questioning the students after their presentations, peers in the TLS group were able to ask pertinent questions of the groups. Presenters were able to respond giving mathematically based explanations and justifications when questioned by students, the teacher or the researcher.

Jason: This is our graph. (The term 'graph' described their table)

Teacher: Can you tell us about that graph?

Jason: We did it because we thought it would make it easier because we were explaining it to Farmer Sprout and we wanted to make it easier so he would know what one to grow it in.

Teacher: So which is the best one to grow it in?

Jason: Sunlight

Teacher: How does your graph back that up?

Jason: Because it shows how much it grew…

Teacher: Why are you saying sunlight? What is it about the table that tells you sunlight?

Hayley: The beans are heavier …

Matthew: Can you tell us on your graph the estimates that you came up with?

Ryan: Well, we counted up like in a pattern and it's going up by a number so we just added the number that it's going up by and added it on. We thought in week 12, it would be that number because it seems to be going up instead of down so we put higher numbers. We put higher numbers as well because we thought it would make it more interesting and if it was going down say – 7,6,5, and we put 15, you wouldn't really get that but if it was going up, we would keep going up as well so that it makes more sense.

However, in the non-TLS group no peers questioned the groups and also when questioned by the researcher, these students had difficulty in answering. The first group had totalled every number in the table and when questioned by the researcher on this, responded with silence after each question. The other group was questioned by the researcher on their prediction.

Tim: Well Shannon, she helped us decide on all different things so we all kind of wrote it.

Researcher: How did she decide?

The whole group, including Shannon was confused and unable to explain their position.

Discussion and Concluding Points

In both groups, it was observed that students focussed on comparing the results represented on the table. However, in doing so there was evidence in the language used by students that the TLS group were aware of the author's comparison structure and this appeared to play a part in their solution planning and written reports. Their oral explanations considered mathematical justifications, but it was not evident if TLS played a role in this. This finding reflects the claims of Bartlett (2003) that TLS can help readers/writers to make sense of problems by identifying relationships within texts. It was interesting to note that the non-TLS peer group did not question any of the groups at all, but each TLS group was questioned mathematically by their peers.

Most groups in both classes embellish their oral and written investigations with prior informal knowledge, such as "Plants need sunlight and rain to grow…" It is interesting to note that the TLS group had just completed a science unit on plants, yet this type of discussion appeared less during their investigations than in the non-TLS groups' investigations. The TLS groups appeared to stay more mathematically on task.

Meyer (2003) reported that knowing how to use text effectively can improve confidence. This seemed to be true throughout this research because there was some evidence that TLS played a significant role in the questioning, explanations and justifications during the presentations. It appears that it had a role in focussing the TLS students on comparing the data and in their ability to engage more in discussion and reporting their findings. While not documented here, there were significant changes in the non-TLS group after they were taught the strategy. While it was significant that it was then their second mathematical-modelling problem, the changes reflected some of the points that were evident in the TLS group after the first problem.

Kiewra (2002) identified structural strategies as a means of teaching students how to learn. This research has proved that young students can be taught to identify structure in texts and that this skill can in some ways make a difference to their mathematical outcomes. Literacy does play a major part in mathematics learning (Cobb, 2004) and evidence here suggests that reading comprehension can effect mathematical modelling outcomes and that TLS can play a relevant part in positively enhancing mathematical-modelling participation. Further in-depth research on the issues raised here is warranted.

References

Bartlett, B. J. (2003). Valuing the situation: a referential outcome for top-level structures. In F. B. B.J. Bartlett, &D. Roebuck (Ed.), *Reimaging Practice: Researching Change* (pp. 16-37). Griffith University, Brisbane: Proceedings of the 1st International Conference on Cognition, Language and Special Education. School of Cognition, Language and Special Education

Bartlett, B. J., & Fletcher, M. (2001). Transforming literacy from secondary to tertiary levels. In F. Bevan, C. Kanes & D. Roebuck (Eds.), *Knowledge demands of the new economy*. Brisbane: Australian Press.

Bartlett, B. J., Barton, B., & Turner, A. (1989). *Knowing what and knowing how: Book 1*. Melbourne: Nelson Educational.

Cobb, P. (2004). Mathematics,literacies, and identity. *Reading Research Quarterly, 39*(3), 333-337.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307-333). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Cobb, P., Confrey, J., diSessa, A., Lesh, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Doerr, H. M., & English, L. D. (2003a). A modeling perspective on students' mathematical reasoning about data. *Journal for Research in Mathematics Education, 34*(2), 110-136.

English, L. D. (in press). Mathematical modeling in the primary school: Children's construction of a consumer guide. *Educational Studies in Mathematics*.

English, L. D. (2004). *Mathematical modelling in the primary school.* Paper presented at the Proceedings of the 27th annual conference of the maths education research group of Australasia Inc, James Cook University, Townsville, Queensland.

English, L. D. (2003a). Mathematical modelling with young learners. In S. J. Lamon, W. A. Parker & S.K.Houston (Eds.), *Mathematical modelling: a way of life* (pp. 3-18). Chichester: Horwood Publishing.

English, L. D., & Lesh, R. (2003). Ends-in-View Problems. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 297-316). Mahwah, New Jersey: Lawrence Erlbaum Associates.

English, L. D., & Watters, J. J. (2005). Mathematical modelling in the early school years. *Mathematics education research journal, 16*(3), 58-79.

Kiewra, K. A. (2002). How classroom teachers can help students learn and teach them how to learn. *Theory into Practice, 41*(2), 71-80.

Lesh, R., & Yoon, C. (2004). What is distinctive in (our view about) models and modeling perspectives on mathematics problem solving, learning and teaching? In H. W. Henn & W. Blum (Eds.), (pp. 151-159): Copywrite by the authors.

Lesh, R., & Doerr, H. M. (2003a). Foundations of a models and modeling perspective on mathematics teaching, learning and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Lesh, R., Zawojewski, J. S., & Carmona, G. (2003). What mathematical abilities are needed for success beyond school in a technology-based age of information. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 205-222). Mahwah, New Jersey: Lawrence Erlbaum

Meyer, B. J. F. (2003). Text coherence and readability. *Topics in Language Disorders, 23*(3), 204-224..

Meyer, B. J. F., & Poon, L. W. (2001). Effects of structure strategy training and signalling on recall of text. *Journal of Educational Psychology, 93*(1), 141-159.

Meyer, B. J. F., Brandt, D. M., & Bluth, G. J. (1980). Use of the top-level structure in text: Key for reading comprehension of ninth-grade students. *Reading Research Quarterly*, 16, 72-103.